

**Decision Analysis  
Technical Report No.82-2**

AD A115964

**Decision Analysis: State of the Field**

by

**Ralph L. Keeney**

March 1982

DTIC FILE COPY

DISTRIBUTION STATEMENT A  
Approved for public release;  
Distribution Unlimited

**Woodward-Clyde Consultants**  
Three Embarcadero Center, Suite 700, San Francisco, CA 94111

**Decision Analysis  
Technical Report No.82-2**

*100014-87-c-0536*

**Decision Analysis: State of the Field**

by

**Ralph L. Keeney**

March 1982

**DTIC  
SELECTED  
JUN 23 1982**

**DTIC**  
Approved for Release  
Distribution Unlimited

**Woodward-Clyde Consultants**  
Three Embarcadero Center, Suite 700, San Francisco, CA 94111

# DECISION ANALYSIS: STATE OF THE FIELD

b<sup>1</sup>

**Ralph L. Keeney**  
**Decision Analysis Group**  
Woodward-Clyde Consultants  
Three Embarcadero Center, Suite 700  
San Francisco, California 94111

This article, written for the non-decision analyst, describes what decision analysis is, what it can and cannot do, why one should care to do this, and how one does it. In the process, we also hope to dispel some myths:

- decision analysis is a tool of operations research and management science,
- some analyses are objective and value-free,
- it would be desirable to have "objective, value-free" analyses,
- decision analysis solves decision problems,
- some decision problems are too difficult for decision analysis,
- decision analysis and decision theory are the same thing.

To accomplish these purposes, we set the stage by describing the decision environment. Then the article presents an overview of decision analysis and provides additional sources for its foundations, procedures, history, and applications.

## 1. THE DECISION ENVIRONMENT

The complexity of the decision environment is greater today than ever. Governmental regulations, such as the National Environmental Policy Act and Occupational Safety and Health Act, require corporations and governmental agencies to consider and justify the impacts. Informed consumers, employees, and shareholders demand greater public consciousness, responsibility, and accountability from corporate and governmental decision makers. For example, executives evaluating potential mergers or acquisitions must consider antitrust and other legal matters, social impacts, and political issues in addition to financial effects. In appraising potential public programs or the elimination of existing programs, a governmental agency should consider not only the multifaceted costs and benefits of its options but also the diversity of the population and its sometimes conflicting viewpoints and political concerns. The bottom line is that today's decision problems are characterized by the following:

*High stakes.* The difference in perceived desirability between alternatives is enormous. It may involve millions of dollars or severe environmental damage, for instance.

*Complicated structure.* Numerous complexities (discussed below) make it extremely difficult to appraise alternatives informally in a responsible manner.

*No overall experts.* Because of the breadth of concerns involved in most important decision problems, there are no overall experts. Different individuals, however, have expertise in disciplines such as economics, engineering, and other professions which should be incorporated into the decision process.

*Need to justify decisions.* Decisions may need to be justified to regulatory authorities, shareholders, bosses, or the public.

Collectively, these characteristics describe many complex decision problems, although the features causing the complexity in specific problems may differ. Because the purpose of analysis is to illuminate complexity and provide insight, it is worthwhile to summarize these features. Although these features are intrinsically intertwined, a categorization is as follows:

1. *Multiple objectives.* It is desirable to achieve several objectives at once. In evaluating routes for proposed pipelines, one wishes simultaneously to minimize environmental impact, minimize health and safety hazards, maximize economic benefits, maximize positive social impact, and please all groups of interested citizens. Unfortunately, all this cannot be done, but it is important to appraise the degree to which each objective is achieved by the competing alternatives.
2. *Difficulty of identifying good alternatives.* Because many factors affect the desirability of an alternative, the generation of good alternatives for careful analysis involves substantial creativity. In some problems, a good deal of soul-searching is required to identify even a single alternative which seems possible, let alone reasonable, for achieving the objectives of the problem.
3. *Intangibles.* How should one assess goodwill of a client, morale of a work force, distress at increasing bureaucracy and governmental regulations, or the aesthetic disruption of a telecommunications tower? Although it is difficult to measure such intangibles, they are often critical factors in a decision.
4. *Long-time horizons.* The consequences of many decisions are not all felt immediately, but often cover (by intention or otherwise) a long time period. For example, the projected lifetime for most major facilities is from 25 to 100 years and research and development projects routinely require 5 to 20 years. Future implications of alternatives now being considered should be accounted for in the decision-making process.
5. *Many impacted groups.* Major decisions, such as constructing canals for crop irrigation or legislation regarding abortions, often affect several groups of people. The impacts to these groups and their attitudes and values differ greatly. Because of these differences, concern for equity contributes to the complexity of a problem.
6. *Risk and uncertainty.* With essentially all problems, it is not possible to predict precisely the consequences of each alternative. Each involves risks and uncertainties—an advertising campaign may fail, a large reservoir may break, a government reorganization may result in an unwieldy bureaucracy, or a new product could turn out to be an Edsel. The major reasons for the existence and persistence of these uncertainties include: (1) little or no data can be gathered for some events, (2) some data are very expensive or time consuming to obtain, (3) natural phenomena such as earthquakes and droughts affect impacts, (4) population shifts affect future impacts, (5) priorities, and hence perceived impacts, change over time, and (6) actions of other influential parties, such as government or competitors, are uncertain.
7. *Risks to life and limb.* A general class of critical uncertainties concerns the risks to life and limb. Numerous personal and organizational decisions affect the likelihood that accidents or "exposure" result in fatalities or morbidity. Examples include decisions about highway maintenance, foods and drugs, toxic or hazardous materials, birth control, leniency toward criminals, and whether to walk or drive somewhere. It is not an easy task to include such dire consequences in an analysis, but it is certainly a part of many decision problems.
8. *Interdisciplinary substance.* The president of a multinational firm can not be professionally qualified in all aspects of international law, tax matters, accounting, marketing, production, and so on. Qualified professionals should supply the relevant inputs on these key factors in a major decision.

9. *Several decision makers.* One player rarely holds all the cards with respect to a major decision. Several players, who may or may not be on the same team, control crucial aspects in the overall decision-making process. To begin production and marketing operations in a new geographical area, corporate management may require approval from stockholders, several regulatory agencies, community zoning boards, and perhaps even the courts. The potential actions of other players must be considered when a corporation evaluates its strategic policy.
10. *Value tradeoffs.* Important decisions involve critical value tradeoffs to indicate the relative desirability between environmental impacts and economic costs today, immediate social costs versus future social benefits, negative impacts to a small group versus smaller positive impacts to a larger group, and sometimes the value of a human life versus the benefits generated by a hazardous technology.
11. *Risk attitude.* A firm operating with the status quo strategy may forecast small and declining profits in the next few years. Changing to an innovative strategy may have a chance of resulting in substantially higher profits, but have a risk of losses or even bankruptcy. Even if the likelihoods of the various consequences are known, crucial value judgments about an attitude toward risk are essential to appraise the appropriateness of accepting risks necessarily accompanying each alternative.
12. *Sequential nature of decisions.* Rarely is one decision completely uncoupled from other decisions. Choices today affect both the alternatives available in the future and the desirability of those alternatives. Indeed, many of our present choices are important because of the options they open or close or the information they provide rather than because of their direct consequences.

Complexity cannot be avoided in making decisions. It is part of the problems, not only part of the solution process. There are, however, options concerning the degree of formality used to address the complexity. Near one extreme, this may be done intuitively in a rather informal manner. Near the other extreme, formal models can be used to capture as much of the complexity as possible. In any case, the process of obtaining and combining the available information is a difficult task that requires balancing all the pros and cons as well as recognizing the uncertainties for each alternative.

## 2. WHAT IS DECISION ANALYSIS

Decision analysis can be defined on different levels. Intuitively, I think of decision analysis as "a formalization of common sense for decision problems which are too complex for informal use of common sense." A more technical definition of decision analysis is "a philosophy, articulated by a set of logical axioms, and a methodology and collection of systematic procedures, based upon those axioms, for responsibly analyzing the complexities inherent in decision problems."

The foundations of decision analysis are provided by a set of axioms stated alternatively in von Neumann and Morgenstern [1947], Savage [1954], and Pratt, Raiffa, and Schlaifer [1964], and the Appendix of this article. These axioms, which provide principles for analyzing decision problems, imply that the attractiveness of alternatives should depend on (1) the likelihoods of the possible consequences of each alternative, and (2) the preferences of the decision makers for those consequences. The philosophical implications of the axioms are that all decisions require subjective judgments and that the likelihoods of consequences and their desirability should be separately estimated using probabilities and utilities respectively. The technical implications of the axioms are that the probabilities and utilities can be used to calculate the expected utility of each alternative and that alternatives with higher expected utilities should be preferred. The practical implication of the decision analysis axioms is the provision of a sound basis and general approach for including judgments and values in an analysis of decision alternatives. This permits systematic analysis in a defensible manner of a vast range of decision problems.

Decision analysis focuses on aspects fundamental to all decision problems, namely

1. a perceived need to accomplish some objectives,
2. several alternatives, one of which must be selected,
3. the consequences associated with alternatives are different,
4. uncertainty usually about the consequences of each alternative
5. the possible consequences are not all equally valued.

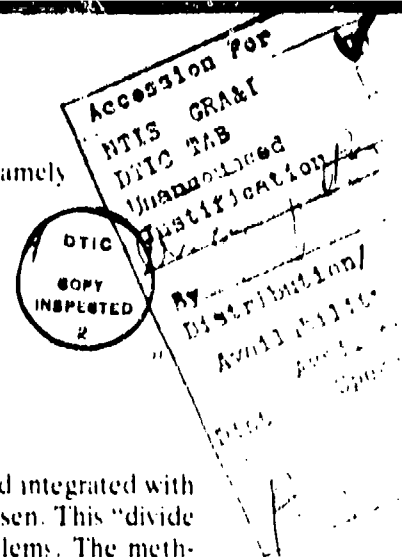
The decision problem is decomposed into parts, which are separately analyzed and integrated with the logic of the decision analysis axioms to suggest which alternative should be chosen. This "divide and conquer" orientation is almost essential for addressing interdisciplinary problems. The methodology of decision analysis provides a framework to combine traditional techniques of operations research, management science, and systems analysis with professional judgments and values in a unified analysis to support decision making. With the procedures of decision analysis, models (e.g., economic, scientific, operations research), available data, information from samples and tests, and experts' knowledge are used to quantify likelihoods of various consequences of alternatives in terms of probabilities. Utility theory is used to quantify the values of decision makers for these consequences.

*Decision analysis is not a tool.* Decision analysis is not simply a tool or a methodology, although it is both sometimes used and perceived as such. Tools of disciplines such as operations research and management science often assume that both the alternatives and objectives of a problem are exogenously provided to the study. Queuing theory, inventory theory, and mathematical programming, for example, concentrate much more on analyzing given alternatives with prescribed objectives. This is of course critically important. However, for less structured decision problems, more time is required to create the alternatives and articulate the objectives before analysis can occur. Decision analysis is designed for such problems. Another important difference which distinguishes tools from decision analysis is the existence of fundamental axioms to provide a philosophically and theoretically sound foundation from which its resulting methodology is developed.

*Objective, value-free analysis is not possible or desirable.* A comment sometimes heard from analysts, governmental authorities, and managers of organizations is that what is really needed to help decision makers is objective, value-free analysis. Simply stated, there is no such thing as objective or value-free analysis. Furthermore, anyone who purports to conduct such an analysis is professionally very naive, stretching the truth, or using definitions of objective and value-free which are quite different from those commonly used. Professional judgments and value judgments are absolutely necessary in essentially every step of analysis in order to address the complexities of decision problems. Objective, value-free analysis would be undesirable because it would simply avoid the problem. What is needed is logical, systematic analysis that makes the necessary professional and value judgments explicit and combines these with the "objective" data for the problem. The resulting analysis should be responsive to the decision maker's needs and justifiable to others. Decision analysis is uniquely a methodology which provides for such analysis.

*Decision analysis is prescriptive in nature.* Prescriptive decision analyses are conducted to indicate which alternative should be chosen to be consistent with the information about the problem and the values of decision makers. This can be contrasted with descriptive studies which attempt to describe how and perhaps why a particular decision was or will be made. Descriptive studies provide useful information (e.g., about cognitive processes on how a competitor might behave) for prescriptive analyses, but by themselves are not prescriptive decision analyses.

It has been clearly demonstrated that individuals often do not make decisions in a manner consistent with the decision analysis axioms (see, for example, Kahneman and Tversky [1979]). However, many of those same individuals find the axioms compelling for prescribing their evaluation of alternatives. The fact is that in complex decision environments, many decision makers prefer to act in accord with the decision analysis axioms and yet seriously violate them in selecting alternatives without the benefit of a decision analysis. This is a strong motivation for the prescriptive appeal of the approach.



*Decision analysis does not solve problems.* Decision analysis will not solve a decision problem, nor is it intended to. Its purpose is to produce insight and promote creativity to help decision makers make better decisions. It does this by providing a methodology and procedures to decompose the problem into parts that can be meaningfully analyzed, a logic to integrate the parts, and documentation for supporting a decision to others. No analysis includes everything of importance in a decision problem. In selecting an alternative the decision makers should jointly weigh the implications of an analysis together with other factors not in the analysis.

This orientation simultaneously implies that decision analysis is not up to the task of solving any decision problem, but that it is appropriate to all. Of course it is not worth analyzing every problem. More difficult decision problems are naturally more difficult to analyze. This is true regardless of the degree to which formal analysis (i.e., use of models as a decision aid) or intuitive appraisal (i.e., in one's head) is used. However, as complexity increases, the efficacy of the intuitive appraisal decreases at a more rapid rate than that of formal analysis. Thus, roughly speaking, it may be more useful to analyze 60 percent of a difficult problem than 90 percent of a simpler problem.

There is another critical factor which relates to the complexity of the decision problem. As complexity increases, the percentage of the problem which can be captured by "hard data" decreases. Simultaneously, the role that values, professional judgment, and experience must necessarily play in the decision process increases. We do not have data bases for the possible consequences of a particular merger, the overall impacts of "rescuing an industry," the "true" probability of low probability-high consequence events, the price and availability of oil in 1990, or the value of the environmental, economic, and social consequences of an oil shale program. Yet decisions involving such factors will necessarily continue and are crucial to everyone. Of all analytical methodologies, only decision analysis provides the theory and procedures to address these directly and incorporate them into a decision problem. It does not provide "the answers" or "the solution," but it does address the right questions.

*Decision analysis and decision theory are not the same.* Broadly interpreted, decision theory is the logical foundations of decision analysis and the technical implications which follow. Decision theory does not include the techniques or skills for structuring decision problems or assessment of probabilities or utilities. The more common interpretation of decision theory is a sampling theory involving statistical problems (see Wald [1950], Savage [1954], and Raiffa and Schlaifer [1961]). This narrow focus of decision theory plus the common misunderstanding that decision analysis and decision theory are essentially the same has led to a misinterpretation about the breadth of problems to which decision analysis is relevant.

### 3. THE METHODOLOGY OF DECISION ANALYSIS

This section presents an overview of the methodology of decision analysis. It is clearly not possible to delve into too much detail. Books by Raiffa [1968], Schlaifer [1969], Tribus [1969], Winkler [1972], Brown et al. [1974], Keeney and Raiffa [1976], Moore and Thomas [1976], Kaufman and Thomas [1977], LaValle [1978], and Holloway [1979] provide more details on various aspects of the methodology. Our purpose is to indicate its general thrust, with emphasis on those aspects unique to decision analysis.

For discussion purposes, the methodology of decision analysis will be decomposed into four steps:

1. structure the decision problem.
2. assess possible impacts of each alternative.
3. determine preferences (values) of decision makers, and
4. evaluate and compare alternatives.

Figure 1 illustrates the interdependencies of the steps and indicates where the components of complexity introduced in Section 1 are addressed. To interpret the implications of these steps, it is

important to keep two facts in mind. First, one iterates among the various steps. Not only what should be done in one step but how it should be done can be affected by preliminary results from another step. Second, decision analyses concentrating on some steps almost to the exclusion of others are often appropriate and useful. Such considerations are mentioned in more detail in Section 4 on the practice of decision analysis.

### Step 1—Structure the Decision Problem

Structuring the decision problem includes the generation of alternatives and the specification of objectives. The creativity required for these tasks is promoted by the systematic thought processes of decision analysis.

Decision analysis captures the dynamic nature of decision processes. It prescribes a decision strategy that indicates what action should be chosen initially and what further actions should be selected for each subsequent event that could occur. For instance, a decision strategy might suggest an initial test market for a new product and then, based on the results, either cancel the product, initiate further testing, or begin a full scale marketing and sales effort. Thus, in describing the alternatives, one must simultaneously specify the decision points, events that may occur between them, and the information that can be learned in the process. This dynamic structure can conveniently be represented as a decision tree (Raiffa [1968]).

Two major problems are associated with generating alternatives. First, there may be a large number of potential alternatives, many of which are not particularly good. However, early in the investigation of the decision problem, it may be difficult to differentiate between the good alternatives and those which are eventually found to be clearly inferior. In such circumstances, inferior options can be identified by screening models which use assumptions too crude for a final evaluation but sensitive enough to weed out the "bad" alternatives. These models analyze a simplified decision problem by using deterministic rather than probabilistic impacts, dominance or "almost dominance" rather than a complete objective function, and constraints. This has the effect of eliminating alternatives so the decision tree is pruned to a manageable size. Then, more time and effort can be expended to carefully appraise the remaining viable alternatives.

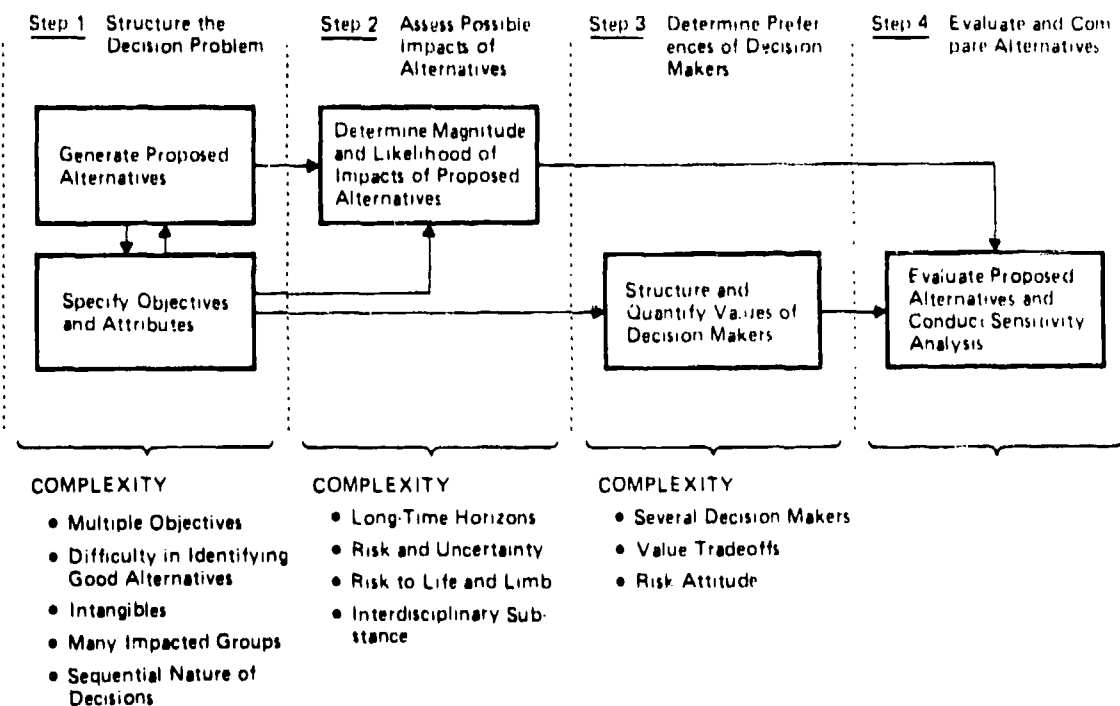


Figure 1. SCHEMATIC REPRESENTATION OF THE STEPS OF DECISION ANALYSIS



A second major problem associated with generating alternatives is that sometimes there seems to be a complete lack of reasonable alternatives. In this case, it is often worthwhile to utilize the objectives of the problem to stimulate creativity. Basically, if the objectives are clearly specified, one can describe possible consequences of the problem which seem particularly desirable. Then one works backward and asks what types of alternatives might achieve such consequences. The process of quantifying the objectives with an objective function (i.e., a utility function as discussed in Step 3) promotes additional thinking about worthwhile alternatives. The result of such a process is often a broadening of alternatives, which is actually a broadening of the decision problem. For instance, a significant increase in local crime may result in a "premature" decision that more police are needed. An analysis may then be initiated of alternatives differing only in the number of additional police. However, the problem is presumably much broader. The objective would likely be to minimize crime or to minimize specific impacts of crime. From this perspective, one may create alternatives involving additional police equipment (e.g., cars, communications), different operating policies with existing personnel and equipment, community action programs to report "suspicious" activity, or the reduction of early release programs for hard-core criminals in jails. A critical change is often the introduction of dynamic alternatives rather than reliance on static alternatives alone. The difference is that a dynamic alternative is designed to be adapted over time based on external circumstances that occur and information that is learned.

The starting point for specifying objectives is the creation of a rather unstructured list of concerns indicating anything of interest about possible consequences of the alternatives. These need to be organized into a set of general concerns. For instance, with many problems involving siting large-scale facilities, the general concerns may be environmental impact, economics, socioeconomic, health and safety, and public attitudes. To determine specific objectives, the question is, for example, what are the environmental impacts of concern for a particular problem. The process of answering such questions is essentially a creative task. However, previous studies on related topics and legal and regulatory guidelines should be of significant help in articulating objectives. Also, for problems which require external review, the potential reviewers (i.e., intervenors, shareholders, or concerned citizens) may contribute useful ideas for objectives.

From all of this information, an objectives hierarchy should emerge with broad objectives pertaining to general concerns at the top of the hierarchy and more detailed objectives further down. The lower-level objectives essentially define the meaning of higher-level objectives; the lower-level objectives are means to the higher-level ends. Gaps in the hierarchy can be identified and filled by following means-ends relationships.

For each of the lowest-level objectives in the hierarchy, we need to identify attributes to measure the degree to which the objective is achieved. Sometimes this is easy. For example, an obvious attribute for the objective "maximize profits" is millions of dollars (why not think big?). However, it is more difficult to determine an attribute for an objective like "minimize visual degradation." This often requires constructing an attribute to measure the objective using procedures such as those in Keeney [1981].

Let us now introduce notation to concisely describe our problem structure. We have generated a number of alternatives  $A_j$ ,  $j = 1, \dots, J$ , and an objectives hierarchy with  $n$  lowest-level objectives  $O_i$ ,  $i = 1, \dots, n$ , where  $n$  may be one. With these lowest-level objectives would be associated attributes  $X_i$ ,  $i = 1, \dots, n$ . Furthermore, define  $x_i$  to be a specific level of  $X_i$ , so the possible impact of selecting an alternative can be characterized by the consequence  $x = (x_1, x_2, \dots, x_n)$ . An example of an objective  $O_i$  is "maximize the local economic benefit" and an associated attribute  $X_i$  may be "annual local tax paid." A level  $x_i$  could then be \$29 million.

The first step of decision analysis addresses several complexities discussed in Section 1. The multiple objective feature is addressed by specifying  $O_1$  to  $O_n$ . Some of these objectives concern the implications to various impacted groups so this feature is also considered. The intangibles are included by using objectives such as "minimize aesthetic disruption" and, of course, significant effort is focused on the complexity of generating viable dynamic alternatives.

## Step 2—Assess Possible Impacts of Alternatives

In this step of decision analysis, we wish to determine the impacts of each alternative. If it were possible to precisely forecast these impacts, we could associate one consequence with each alternative. Then the evaluation of alternatives would boil down to a choice of the best consequence. Unfortunately, the problem is usually not so easy because of uncertainties about the eventual consequences. Therefore, for each possible alternative, it is desirable to determine the set of possible consequences and the probabilities of each occurring. This can be done formally by determining a probability distribution function  $p_i(x)$  over the set of attributes for each alternative  $A_i$ . In some cases the uncertainty associated with an alternative may be small. Then, an appropriate simplification is to omit the uncertainty for that alternative. Because one can treat  $p_i$  in general to include cases with no uncertainty (where  $p_i(x)$  assigns a probability one to a particular  $x$  and zero to all others), we will use  $p_i$  throughout.

When feasible, meaning that both general knowledge about the problem structure and the scope of the project allows it, it is desirable to determine probabilities of possible consequences with the development and use of formal models. These models typically utilize the traditional methodologies of operations research, management science, systems analysis, simulation, planning, and the sciences and engineering. Complex models can often be constructed to have several components, each pertaining to knowledge associated with a single discipline or organizational unit. For instance, in a decision analysis by Smallwood and Morris [1980] to examine whether to build a new manufacturing facility, a model had components concerning the market for the proposed product, maintenance, production, capital costs, the competition, and the financial impact to the company. Experts in each of these substantive areas could then provide information on their respective part of the problem. Hence, these models allow one to break the assessment into manageable parts and combine the parts to determine  $p_i$ .

When a model is utilized, either deterministic or probabilistic information is required to specify model inputs in order to determine appropriate probability distributions over model outputs (i.e., consequences). When a model is not appropriate, information is necessary to directly determine possible consequences. In both cases, such information must be based on the analysis of existing data, data collected specifically for the decision problem, or professional judgment. Data analysis is common to many disciplines other than decision analysis so, although it is important, it will be passed over here. The quantitative assessment of professional judgments or probabilities is a unique aspect of decision analysis discussed below.

There are several methods for quantifying probabilities (see Winkler [1967a], Spetzler and Staël von Holstein [1975], and Staël von Holstein and Matheson [1979]). One method is to use a standard probability distribution function and assess parameters for that function. For example, the parameters of a normal distribution can be the mean and standard deviation. Another technique, referred to as a fractile method, involves directly assessing points on the cumulative probability density function. Suppose  $y$  is the single dimensional parameter of interest and we wish to assess the probability density function  $p(y)$ . One is asked for a level  $y'$  such that the probability is  $p'$  that the actual level is less than  $y'$ . This questioning is repeated for several probabilities such as  $p' = 0.05, 0.25, 0.5, 0.75$ , and  $0.95$ . Alternatively one can ask for a probability  $p''$  that the  $y$ -level is less than  $y''$ . By fitting a common probability distribution to the assessed data, one obtains  $p(y)$ . A third procedure for assessment is appropriate when the possible impact is categorized into a number of distinct levels. The professional familiar with the subject is asked to specify directly the probability of each level. These assessments may sound easy, but in practice they are involved processes with many potential sources for error (see, for example, Tversky and Kahneman [1974, 1981]). However, recent experience suggests that professionals with training can formulate probabilistic forecasts in a reliable manner (see Murphy and Winkler [1977]).

A factor which can increase the complexity of impact assessments is probabilistic dependencies among attributes for given alternatives. If two attributes are probabilistically dependent, the impact specified for one will affect the assessed impact on the other. When there are such conditional dependencies, it is almost essential to either model these dependencies and develop probabilistic assessments using the output of the model or to bound the possible probability distributions utilizing logic and understanding of the problem (see, for example, Sarin [1978] and Kirkwood and Pollack

[1980]). Then one can investigate whether and how the dependencies influence the evaluation of alternatives. When such dependencies turn out to be important, additional effort to better characterize them may be appropriate.

A host of additional difficulties can occur when more than one expert is asked for professional judgments about the same events. These experts may have different opinions, and yet it may be almost impossible to find out the reasons for the differences. And the experts likely formulate their judgments based in part on the same experiments and data sources, so they are not independent. Still, the decision maker may desire a single coherent representation of the uncertainty in the problem. Recent contributions by Morris [1977] and Winkler [1981] address this problem, which is one area of current research in decision analysis.

Specifying probability distributions addresses the risk and uncertainty aspects of the decision problem. In describing the possible impacts, the time in which consequences might occur should be indicated. Thus, the feature of long-time horizons is addressed in this step. The interdisciplinary substance is also included by utilizing the skills of the various disciplines to develop and structure models, provide information and professional judgments relevant to the discipline, and appraise the results of the model about possible consequences concerning the disciplinary substance.

### Step 3—Determine Preferences (Values) to Decision Makers

It would likely be impossible to achieve the best level with respect to each objective in a decision problem. The question is, "How much should be given up with regard to one objective to achieve a specified improvement on another?" The issue is one of value tradeoffs. For decision problems with either single or multiple objectives, it is rarely the case (except in simple problems) that one alternative is guaranteed to yield the best available consequence. There are usually circumstances that could lead to undesirable consequences with any given alternative. The question is, "Are the potential benefits of having things go right worth the risks if things go wrong?" This issue is about risk attitudes. Both value tradeoffs and risk attitudes are particularly complicated because there are no right or wrong values. Basically, what is needed is an objective function which aggregates all the individual objectives and an attitude toward risk. In decision analysis, such an objective function is referred to as a utility function, symbolically written  $u$ . Then  $u(x)$ , the utility of the consequence  $x$ , indicates the desirability of  $x$  relative to all other consequences. As mentioned in Section 2, following directly from the axioms of decision analysis, alternatives with higher expected (i.e., average) utilities should be preferred to those with lower expected utilities.

This step, rather unique to decision analysis, involves the creation of a model of values to evaluate the alternatives. This is done in a structured discussion with the decision makers to quantify value judgments about possible consequences in the problem. The procedure systematically elicits relevant information about value tradeoffs, equity concerns, and risk attitudes with provision for consistency checks. In addition to the obvious advantage of providing a theoretically sound manner to evaluate alternatives, the explicit development of a value model offers several other advantages, including indicating which information is of interest in the problem, suggesting alternatives that may have been overlooked, providing a means to calculate the value of obtaining additional information, and facilitating concise communication about objectives among interested parties. In addition, a sensitivity analysis of the value judgments can be conducted to appraise their importance for the overall decision.

The process of determining the utility function can be broken into five steps: (1) introducing the terminology and ideas, (2) determining the general preference structure, (3) assessing single-attribute utility functions, (4) evaluating scaling constants, and (5) checking for consistency and reiterating. For decision problems with a single objective, only Steps 1, 3, and 5 are relevant. In practice there is considerable interaction among the steps although each will be separately discussed.

*Introducing the Terminology and Ideas.* The basic purpose of this step is to develop a rapport and an ability to communicate with the decision maker or decision makers. It should be stated that the goal of the assessment process is to end up with a consistent representation of preferences for evaluating alternatives. The analyst should make sure that the decision makers are comfortable with the assessment procedure and understand the meaning of each attribute and the objective it is

meant to measure. If the decision makers have not been closely involved in defining the attributes or describing the impacts of alternatives, this phase of communication is particularly important. The decision makers should understand that there are no correct or incorrect preferences and that expressed preferences can be altered at any time.

*Determining The General Preference Structure.* Here, one structures preferences with a model indicating the general functional form of the utility function  $u(x_1, \dots, x_n)$ . To obtain the structure for multiple objectives, one uses value independence concepts in the same way that probabilistic independence is utilized in structuring models of impacts. Most of the independence concepts concern relative values for consequences with levels of a subset of the attributes fixed. The independence concepts are used to derive a simple function  $f$  such as

$$u(x_1, \dots, x_n) = f[u_1(x_1), \dots, u_n(x_n), k_1, \dots, k_m, \dots, k_R] \quad (1)$$

where the  $u_i$  are single-attribute utility functions and the  $k_m$  are scaling constants. Specific functional forms following from various assumptions are found in Fishburn [1964, 1965, 1970], Meyer [1970], Farquhar [1975], Keeney and Raiffa [1976], Bell [1977b, 1979b], Tamura and Nakamura [1978], and Farquhar and Fishburn [1981]. Using (1), the overall utility function is determined by assessing the single-attribute utility functions and the scaling constants which weight various combinations of single-attribute functions.

A related approach to model values for multiple objectives involves building a value function  $v(x_1, \dots, x_n)$ , which assigns higher numbers (i.e., values) to preferred consequences. This is done in a spirit akin to (1) using either single-attribute value functions or indifference curves together with scaling constants. Then a utility function is assessed over value providing  $u[v(x)]$  which incorporates value tradeoffs in  $v$  and an attitude toward risk in  $u$ . Models of value functions addressing multiple objectives are found in Debreu [1960], Koopmans [1960], Luce and Tukey [1964], Krantz [1964], Krantz et al. [1971], Dyer and Sarin [1979], Kirkwood and Sarin [1980], and Keelin [1981]. A commonly used value function is discounting of cash flows over time at a fixed rate. Boyd [1973] and Keeney and Raiffa [1976] discuss procedures to obtain both  $v(x)$  and  $u(v)$ .

*Assessing Single-Attribute Utility Functions.* Procedures for assessing single-attribute utility functions are well developed. In summary, one wishes to first determine the appropriate risk attitude. For instance, for consequences involving profits, one is said to be risk averse if profit level  $(x_1 + x_2)/2$  is always preferred to a lottery yielding either  $x_1$  or  $x_2$  each with a probability of 0.5. In this case, one prefers the average of the profits  $x_1$  and  $x_2$  for sure rather than risk a half chance of the higher and a half chance of the lower. When one is risk averse, it must be the case that the corresponding single-attribute utility function is concave. As discussed in Pratt [1964], special risk attitudes restrict the functional form of single-attribute utility functions. A common utility function is the exponential utility function

$$u(x) = a + b^{-cx} \quad (2)$$

where  $a, b > 0, c > 0$  are scaling constants. This utility function is referred to as constantly risk averse since it is the only one consistent with the following property. If  $x_3$  for sure is indifferent to a 0.5 chance at either  $x_1$  or  $x_2$ , then  $x_3 + \epsilon$  must be indifferent to 0.5 chances at either  $x_1 + \epsilon$  or  $x_2 + \epsilon$  for all possible  $\epsilon$ .

To specify the scaling constants  $a$  and  $b$  in (2), one arbitrarily sets the utility corresponding to two consequences. This is similar to defining a temperature scale by selecting a boiling and a freezing point. The utilities of all other consequences are relative to the two chosen for the scale. To specify the appropriate numerical value for a constant  $c$  in (2), one can identify both a lottery and a consequence which are equally preferred by the decision maker. For instance, suppose the decision maker is indifferent between the certain consequence  $x_3$  and a lottery yielding either  $x_1$  or  $x_2$  with equal chances of 0.5. Then, to be consistent with the axioms of decision analysis, the utility of  $x_3$  must be set equal to the expected utility of the lottery. Hence,

$$u(x_3) = 0.5u(x_1) + 0.5u(x_2). \quad (3)$$

Substituting (2) into (3) and solving gives us the value for parameter  $c$ .

*Evaluating Scaling Constants.* With multiple objectives, the same concept is utilized to determine scaling constants, which relate to the relative desirability of specified changes of different attribute levels. To illustrate this in a simple case, consider the additive utility function

$$u(x_1, \dots, x_n) = \sum_{i=1}^n k_i u_i(x_i). \quad (4)$$

where  $k_i$ ,  $i=1, \dots, n$  are scaling constants. For this additive utility function, the values of the  $k_i$  indicate the relative importance of changing each attribute from its least desirable to its most desirable level. To assess these scaling constants, one generates data representing stated value judgments of the decision maker. For instance, the decision maker may be indifferent between  $(x_1, \dots, x_n)$  and  $(x_1, \dots, x_n)$ . Then the utility of these two consequences, since they are indifferent, must be equal. They are set equal using (4) which yields an equation with the scaling factors as unknowns. Using such indifferences, one generates a set of  $n$  independent equations which is solved to determine values for the  $n$  unknown scaling factors. The equations can be generated by sequentially considering consequences which differ in terms of the levels of only two attributes. This significantly simplifies the comparison task required of the decision makers. More details about the assessment of utility functions can be found in Fishburn [1967], Huber [1974], Keeney and Raiffa [1976], Bell [1979a], and many other sources.

*Checking consistency.* It has been my experience that invariably there are inconsistencies in the initial assessments. In fact, this is one of the main reasons for the procedure, because once these inconsistencies are identified, decision makers upon reflection can alter some of their responses to reach consistency and better reflect their basic values. Furthermore, they seem to feel better after having straightened out their value structure in their own mind. Thus, it is essential to ask questions in different ways and carefully reiterate through aspects of the assessment procedure until a consistent representation of the decision maker's values is achieved. Conducting sensitivity analysis of the evaluation of alternatives (Step 4 of decision analysis) may suggest if the utility function is a good enough representation of decision maker values.

With multiple decision makers, as discussed in Harsanyi [1955], Fishburn [1973], or Keeney and Raiffa [1976], additional value judgments are required to address the relative importance of the different decision makers and the relative intensity of the potential impact to each in order to determine an overall utility function. Alternately, the decision problem can be analyzed from the viewpoints of the different decision makers by using their own utility functions. It may be that the same alternative is preferred by each decision maker, possibly for different reasons. In any case, it might be helpful to eliminate dominated alternatives, identify the basis for conflicts, and suggest mechanisms for resolution.

This third step of decision analysis uses value judgments to address the complexities concerning value tradeoffs, and a risk attitude outlined in Section 1. The value judgments are made explicit in assessing  $u$  for each decision maker. This process of building a model of values corresponds precisely with that used for any model. We gather some data (the decision maker's judgments), and use the data in a generic model (the utility function  $u$ ) to calculate its parameters (e.g., the  $k_m$ 's in (1) and  $c$  in (2)). Additional value judgments are necessary to structure values of multiple decision makers into one coherent utility function.

#### Step 4—Evaluate and Compare Alternatives.

Once a decision problem is structured, the magnitude and associated likelihoods of consequences determined, and the preference structure established, the information must be synthesized in a logical manner to evaluate the alternatives. It follows from the axioms of decision analysis that the basis for this evaluation is the expected utility  $E_j(u)$  for each alternative  $A_j$ , which is

$$E_j(u) = \int p_j(x)u(x)dx. \quad (5)$$

The higher  $E_j(u)$  is, the more desirable the alternative. Thus the magnitudes of  $E_j(u)$  can be used to establish a ranking that indicates the decision maker's preferences for the alternatives. It should be remembered that the expected utility associated with an alternative is directly related to the objectives originally chosen to guide the decision and reflects the degree of achievement of the objectives. One can transform the  $E_j(u)$  numbers back into equivalent consequences to obtain information about how much one alternative is preferred over another.

It is extremely important to examine the sensitivity of the decision to different views about the uncertainties associated with the various consequences and to different value structures. This is conceptually easy with decision analysis, since the impacts and values are explicitly quantified with probability distributions and the utility function, respectively. Without quantification it would be

difficult to conduct a thorough sensitivity analysis. A useful way of presenting the results of a sensitivity analysis is to identify sets of conditions, in terms of uncertainties and preferences, under which various options should be preferred.

#### 4. PRACTICE OF DECISION ANALYSIS

The ultimate purpose of decision analysis is to help decision makers make better decisions. The foundations, provided by the axioms do not "assume the problem away." Even though the theory and procedures are straight-forward, a price is paid for attempting to address the complexities of a decision problem explicitly. The implementation phase, that is putting the methodology into practice, is more involved compared to other forms of analysis. A significantly greater portion of the overall effort in decision analysis is spent generating alternatives, specifying objectives, eliciting professional and value judgments, and interpreting implications of the analysis. Each of these requires interaction with the decision makers and individuals knowledgeable about the problem substance. Structured creative thinking is demanded and sensitive information is elicited.

In this section, we suggest how to conduct a decision analysis and the art of interaction necessary to elicit information. Several uses of decision analysis in addition to evaluating alternatives are indicated. Finally some key potential pitfalls are identified.

##### Conducting a Decision Analysis

A careful definition of the decision problem is essential. For complex problems, an adequate definition is rarely available at the time the analysis is to begin. Yet, it is tempting to begin analyzing the problem immediately. What is available at the *beginning* is a somewhat vaguely perceived notion of problem objectives and possible alternatives. Defining a problem means the following: generating specific objectives with appropriate attributes and articulating dynamic alternatives including possible information to be learned in the decision process. The attributes indicate what information is wanted about the alternatives, namely the degree to which the alternatives measure up in terms of the attributes.

If the utility function is assessed to quantify the decision maker's values, this will indicate the relative importance of gathering different information. That is, Step 3 of a decision analysis can proceed before Step 2 (see Figure 1). This is often useful because structuring a decision problem and assessing values require only personal interaction which is much less expensive than field tests, equipment, and surveys often necessary to quantify the impacts of the alternatives. Knowing what information to collect may reduce this burden or at least focus it on the information desired. One other point is worth mentioning in this regard. There is one value structure for a decision problem since each alternative is to achieve the same objectives. There are possible impacts to be assessed for each alternative. Thus, concentrating on the values first and thoroughly may save time, effort, and money on a decision analysis, and result in more useful insights for the problem.

Once the decision problem is well-structured, the collection of information should proceed as indicated in Step 2 of Section 3. The process may be complicated because of problem substance or required personal interaction. The former situation is not unique to decision analysis and will not be discussed further.

##### The Art of Decision Analysis Interaction

A key to successful decision analysis is the interaction of decision analysts with the decision makers and other professionals working on the project. As with all forms of personal interaction, there is a great deal of art and skill required. Most of the skills required to be a successful member of any group are also necessary to be a successful member of the team analyzing the decision process. However, because of the nature of decision analysis, we will note a few special aspects related to that interaction process.

Decision analysts obtain clearly articulated (often quantitative) information about the problem structure, possible impacts, parameters for a model, and value judgments. In addition to the complexity of the problem substance, the practice of obtaining such information is sometimes difficult because:

1. the information may be sensitive,
2. the natural procedures to process the information in one's mind often result in biased judgments,
3. the respondent may have a vested interest in misrepresenting information.

The decision analyst should be aware of any of these three possibilities.

In a recent article, Fischhoff [1980] draws an analogy between decision analysis and psychotherapy. Decision analysts try to formalize the thinking and feelings that the decision maker wishes to use on the problem. By clarifying and even quantifying the process, these thoughts and feelings are potentially opened for review by others (e.g., bosses, regulators, courts). In any assessment process, one should take the time and use any available devices to establish a rapport with the respondent and to make him or her feel comfortable. I always point out that the reason for the analysis is that the problems are too difficult to informally analyze consistently. Hence, a major purpose of these processes is to identify inconsistencies in the unassisted thinking of the respondent. It is critical to assure these individuals that they will have a first right to adequately review your work. Furthermore, they should have the option of changing their responses. This helps to ensure that no misrepresentation of their judgments occurs. What this boils down to is the need to build trust between the decision analyst and all respondents working on a decision problem. The establishment of this trust must be the responsibility of the decision analyst.

Tversky and Kahneman [1974, 1981] have identified many biases that individuals may inadvertently utilize in providing professional or value judgments. It is probably safe to say that these biases occur with any procedure, formal or informal, to assist in the decision making process. With decision analysis which focuses on such issues, reasonable procedures have been developed with enough consistency checks to avoid or at least identify the major biases which may be influencing the particular analysis. Many professionals, including Winkler [1967b], Slovic and Lichtenstein [1971], Hogarth [1975], Spetzler and Staël von Holstein [1975], Fischer [1976, 1979], Seaver, Edwards, and von Winterfeldt [1978], and Alpert and Raiffa [1981], have compared various approaches to examine their strengths and weaknesses for such assessments.

A more difficult issue for the analyst might be that of potential conflict. A decision maker who wishes that a particular product be produced and marketed may be motivated to overestimate its potential sales. A product manager being evaluated on meeting a specific goal may care to underestimate the potential sales during the goal setting process. To assist in identifying such conflicts, aside from one's knowledge of the position of individuals with respect to the problem, several techniques are used to reduce the conflicts.

Effects due the sensitive nature of decision information, inherent conflicts, and unconscious biases, can be reduced by using four devices: iteration with consistency checks, assessments with different individuals, decomposition, and sensitivity analysis. Information should be gathered using redundant lines of questioning, and resulting inconsistencies should be investigated until consistency is achieved. Then, there is some comfort that the major discrepancies are eliminated. Use of judgments about the same factor obtained from different qualified individuals has obvious virtues. Decomposition involves dividing the assessment into component parts and obtaining judgments on the components. For instance, in addition to asking the product manager about profit from the product, ask component judgments about product manufacturing costs, distribution costs, potential sales at various prices, pricing policy, and competitor actions. Different individuals should provide these inputs which would then be utilized to provide estimates of profit. Sensitivity analysis can identify problem elements which are crucial for the evaluation of the alternatives. It is only for these that significant effort is necessary to appraise the recommendations of the analysis.

## Uses of Decision Analysis

As previously mentioned, the overall use of decision analysis is to provide insight to improve decision making. One key manner of deriving this insight is to evaluate the alternatives. This is of course common to most prescriptive analytical approaches. However, decision analysis has other crucial uses to provide insight.

A strength of decision analysis is that one can readily calculate the value of additional information (see LaValle [1968] and Merkhofer [1977]). This is done by defining and evaluating alternatives which include the costs of gathering specific information and the likelihoods of what that information will be. For example, a test market for a proposed new product may cost one million dollars and the results may indicate potential annual sales anywhere between 20,000 and 500,000 sales per year. If the "test market" alternative has a higher expected utility than the "no test market" alternative, it is worthwhile. By raising the cost of the test market, we can find the cost where these two alternatives are indifferent. This cost is referred to as the value of the test market information and indicates the maximum one should pay for that information. Using this basic idea, Gilbert and Richels [1981] analyze the value of uranium resource information for U.S. energy policy decisions.

Because of the focus on problem complexities, there are many useful byproducts of decision analysis. The framework of decision analysis promotes honesty by providing the opportunity for various independent checks and centers communication on crucial problem features. For instance, one often develops a clear understanding of the substantive issues of a problem in the process of structuring the objectives hierarchy. This also has the effect of sensitizing different individuals to the issues and perhaps bringing about a commonality of understanding of what the problem is or at least a common set of terms to discuss the problem. Also, creative alternatives can be generated by stimulating thinking based on the problem objectives.

Finally, decision analysis can be very important in conflict identification and resolution. It should indicate whether conflicts among various individuals concern the possible impacts or the values for these impacts. Furthermore, conflicts may only involve certain objectives in either case. Once conflicts are identified, attention can be concentrated on their resolution by examining the bases for judgments of each individual concerned. It may be that only parts of the individuals' bases differ and these parts are the reason for the conflict. Information might be gathered which would resolve such a conflict. However, there are irresolvable conflicts, such as justifiable differences in values. For these cases, identification of the basis for the conflict may in itself be an important contribution toward a more responsible decision.

Many decision analyses do not need to be complete. Partial decision analyses which give cursory qualitative attention to some steps in Section 3 are definitely appropriate for many decision problems. These partial analyses should focus on the aspects of the overall problem where insight might be most fruitful to the decision makers. Once the problem is structured or the impacts of alternatives clarified or the values articulated, the rest of the analysis may be easy or even unnecessary. In these partial analyses, the unique contribution of decision analysis is often the procedures to address explicitly the softer parts of the problem—its structure and professional and value judgments.

## Pitfalls of Decision Analysis

Decision analysis is subject to the same pitfalls as other approaches designed to assist decision makers. One might categorize these pitfalls as follows:

1. weak or no logical or theoretical foundations.
2. lack of consideration of subjective and value components of the decision problem.
3. a claim that analysis provides a solution to the decision problem.
4. poor analysis.
5. weak personal interaction skills.



As previously stated, the foundations of decision analysis are strong and the subjective and value aspects of the decision problem are addressed. Hence, specific pitfalls under categories 1 and 2 are rarely the downfall of a decision analysis.

Category 3 represents a pitfall often more common to decision analysis than other approaches. Because decision analysis does try to capture a bigger share of the "real problem," there is a tendency to assume the entire problem is addressed. Worse though is the misrepresentation that such an analysis provides a solution to the decision problem. Decision analysis, indeed any analysis, only focuses on part of a problem and this should be understood.

Poor analysis or poor personal interaction can, of course, render the best conceived decision analysis as worthless. Rather than repeat all the things that could go wrong here, it may be more appropriate to refer to Keeney and Raiffa [1972] for a short critique of decision analysis or to Majone and Quade [1980] for an entire volume on pitfalls of analysis.

## 5. APPLICATIONS OF DECISION ANALYSIS

Discussions of early applications of decision analysis in the oil and gas industry are described in Grayson [1960] and Kaufman [1963]. Applications also occurred in other fields. However, for proprietary reasons, many of the completed decision analyses do not appear in the published literature. Fortunately, the essence of some of these analyses do appear in the form of "fictitious" analyses or case studies. Magee [1964a,b] describes applications to capital investment decisions. Howard [1966] discusses a product introduction, and a number of cases representing experiences of the early 1960's are found with analyses in Schlaifer [1968]. Papers by Brown [1970] and Longbottom and Wade [1973] surveyed applications of decision analysis through the 1960's.

The 1970's saw an expansion in applications of decision analysis. The applications concerned both private industry and governmental decisions. They involved new product decisions, research and development efforts, medical problems, energy problems, environmental alternatives, and standard setting, to name a few. In this article, it would not be possible to survey all of these applications. Hence we will simply attempt to indicate sources of some applications which are readily available. Many of these sources describe other applications.

There have been many applications of decision analysis addressing various corporate problems. Although many of these are proprietary, there are some published examples of these corporate decision analyses. Spetzler [1968] describes the procedure of assessing a utility function for a corporate board. Matheson [1969] summarizes an application concerning the introduction of a new product. The book by Brown, Kahr and Peterson [1974] describes several applications. Keeney [1975] discusses the assessment of a multiple objective corporate utility function to examine corporate policies. Keefer and Kirkwood [1978] discuss an application to optimally allocate an operating budget for project engineering. A recent application described in Smallwood and Morris [1980] considers whether Xerox Corporation should construct new manufacturing facilities for a new product and when this should be done. Stillwell et al. [1980] report the evaluation of credit applications.

Rather than discuss selected applications in medical fields, it is simpler to refer readers to a recent annotated bibliography of decision analysis applications by Krischer [1980]. The applications address such diverse problems as evaluating governmental programs to save lives, the evaluation of new drugs, the selection of medical technologies for advanced medical systems, analyses to select treatment strategies for numerous different diseases or ailments, the development of on-line computer systems to assist physicians in decision making, and the development of various health industries.

There have been numerous applications of decision analysis to problems faced by various branches of government over the last decade. Examples of these include the possibility of seeding hurricanes threatening the coasts of the United States (Howard et al. [1972]), metropolitan airport development in Mexico City (de Neufville and Keeney [1972]), protection from wildland fires (North et al. [1975]), trajectory selection for the Mariner Jupiter/Saturn project (Dyer and Miles [1976]), and the evaluation of busing alternatives to achieve school integration (Edwards [1980]). Several more recent applications of decision analysis to governmental problems concern selection of stan-

dards. Examples are emission control strategies by North and Merkhofer [1976], chronic oil discharge standards by von Winterfeldt [1982], and the negotiation of international oil tanker standards by Ulvila and Snider [1980].

By their nature, significant environmental problems concern both government and industry. In the recent past, there have been a large number of decision analyses addressing such environmental problems. Examples are the work of Gardiner and Edwards [1975] concerning development within the areas under the jurisdiction of the California Coastal Commission, work involving Bell [1977a] and Holling [1978] concerning control of a forest pest, the analysis of marine mining options by Lee [1979], and the evaluation of regional environmental systems by Seo and Sakawa [1979].

The area with perhaps the greatest number of applications in recent years has been energy. There have been decision analyses of the United States synthetic fuels policy (Synfuels Interagency Task Force [1975]) and nuclear reactor program (Manne and Richels [1978]), expansion of the California electrical system capacity (Judd [1978]), management of nuclear waste (Lashrop and Watson [1982]), and commercialization of solar photovoltaic systems (Boyd et al. [1982]). There has been considerable effort focused on alternatives faced by the utility industry. These include the selection of technological alternatives for specific projects such as transmission conductors (Crawford et al. [1978]), the examination of the implications of both over- and under-capacity (Cazalet et al. [1978]), the siting of energy facilities (Keeney and Nair [1977], Keeney [1980b], and Sarin [1980]), and the choice between coal and nuclear technology for large-scale power plants (Beley et al. [1981]).

## 6. HISTORY OF DECISION ANALYSIS<sup>1</sup>

It is difficult to trace decision analysis from its beginning to the present because of the evolutionary nature of both its content and its name. The foundations of decision analysis are the intertwined concepts of subjective probability and utility, and Ramsey [1931] was the first to suggest a theory of decision making based on these two ideas. Two centuries earlier, Bernoulli [1738] wrote a remarkable paper on the motivation for the concept of utility and on a possible form for a utility function. For a historical discussion of the early development of subjective probability and utility theory, see Fellner [1965]. On the uncertainty side, DeFinetti [1937] contributed greatly to the structure of subjective probability. Modern utility theory for decision making under uncertainty was developed, independently, by von Neumann and Morgenstern [1947]. They postulated a set of axioms similar to those in the Appendix (using only objective probabilities) and demonstrated that a utility could be assigned to each consequence in a manner such that the decision maker should prefer the alternative with the highest expected utility in order to act in accord with the axioms. This result is often referred to as the expected utility hypothesis.

Wald [1950], in his classic work on statistical decision problems, used theorems of game theory to prove certain results in statistical decision theory. Although he used an expected-loss criterion instead of utility theory, it was only a minor modification to introduce utility into the Wald framework. This work highlighted a critical problem, namely, how to account for informal information about the states of the world in his model. The school of statisticians and decision theorists, including J. Marschak, H. Chernoff, and H. Rubin, advocated the use of judgmental probability as one method of tackling the statistical decision problems proposed by Wald. The pioneering work of Blackwell and Girshick [1954] contributed to the integration of utilities and subjective probabilities into a coherent program for handling these problems. Then Savage [1954], in a major contribution, provided a rigorous philosophical foundation and axiomatic framework for the approach.

Once the theory was developed, many individuals began applying it to mathematically well-structured problems involving uncertainties and possibilities for sampling or experimentation. These results, building on the previous work of others, formed a body of results known as Bayesian or statistical decision theory (Schlaifer [1959], Raiffa and Schlaifer [1961], Pratt et al. [1965]). When in the early 1960's these same individuals and their associates, mainly at the Harvard Business School, began using these theories on real business problems involving uncertainties, whether or

<sup>1</sup>This section is liberally adapted from Keeney [1978].

at sampling or experimentation were possible, an adjective was added to yield applied statistical decision theory. However, since applied statistical decision theory was relevant to broad classes of complex decision problems (see Schlaifer [1969]), it was better to have a more application-oriented name, and the term decision analysis appeared in the literature (Howard [1966]).

Over the past thirty years, the contributions of many people concerned with the behavioral aspects of decision making have had a significant impact on prescriptive decision analysis. Mosteller and Nogee [1951], Friedman and Savage [1952], Edwards [1954], and Davidson et al. [1957] made early contributions to the assessment of preferences and judgments. Excellent sources of this work are Slovic and Lichtenstein [1971], Tversky and Kahneman [1974, 1981], Hammond et al. [1980], and Einhorn and Hogarth [1981].

Procedures have been developed to better account for specific characteristics of decision problems mentioned in Section 1. Examples include work by Pratt [1964] and Schlaifer [1969] on assessing utility functions; by Winkler [1969, 1981], Edwards [1968], Schlaifer [1969], Spetzler and Staël von Holstein [1975], and Morris [1977] on assessing probability distributions; by Arrow [1963], Harsanyi [1955] and Keeney and Kirkwood [1975] on group preferences; by Fishburn [1964, 1965, 1974], Pollak [1967], Raiffa [1969], and Boyd [1973] on multiattribute preferences; by Koopmans [1960], Lancaster [1963], Meyer [1977], and Bell [1977b] on preferences over time; by Rousseau and Matheson [1967], Schlaifer [1971], Keeney and Sicherman [1976], and Seo et al. [1978] on developing software systems for structural and computational assistance; by Miller et al. [1976], Jungermann [1980], and von Winterfeldt [1980] on structuring decision problems; and numerous contributions of people in statistics, stochastic processes, systems analysis and computer science to develop better probabilistic models. In many analyses concerning, for example, liquefied natural gas, nuclear power, and hazardous wastes, a critical issue is the value of the lives which may be lost due to either an accident or "normal" use. Some of the current methodological development in decision analysis concerns critical value judgments such as the value of human life (see, for example, Bodily [1980], Howard [1979], Keeney [1980a], and Pliskin et al. [1980]).

## 7. RESEARCH

The techniques and procedures of decision analysis are sufficiently developed to make substantial contributions on many complex decision problems. Compared to other approaches, both formal and informal, decision analysis often has much to offer. Compared to "providing all the insight a decision maker could possibly want at a price too low to refuse," there are significant improvements which could be made. Research on the following topics will help lead to these improvements. This research is categorized by the steps of decision analysis outlined in Section 3.

Regarding structure of the decision problem, better approaches to develop objective hierarchies, create alternatives, and interrelate the two are needed. The approaches should be systematic to increase the likelihood that important parts of the problem are not being omitted. Procedures are needed to help identify relevant stakeholders—individuals or groups with an interest in the problem—and ensure that their objectives and advocated alternatives are recognized by the analysis. Likewise, more systematic procedures are needed to identify exogenous events not under the decision makers control which could significantly influence the consequences of alternatives. This should reduce the likelihood of unanticipated consequences. Any analysis excludes many aspects felt to be less relevant than included ones. Yet, many decision problems are interrelated even though at some level it is impractical to include these interrelationships in detail in an analysis. Better means to address the interrelationships are needed in decision analysis. As a specific example, a carefully defined attribute to measure flexibility in a decision problem might address the degree to which alternatives in a given problem might foreclose options in other decision problems.

With possible impacts, two major problems deserving additional research concern probabilistic dependencies and multiple experts. Better methods to identify probabilistic dependencies and to elicit subjective probability distributions with probabilistic dependencies would be helpful. On many important problems, different experts disagree about the impacts expected from various alternatives. Research is needed to provide methods to reduce such discrepancies when appropriate

and to produce responsible representations of the "collective judgment" in cases where discrepancies persist.

Models of values could be improved in three significant respects. First, models to better characterize the values of a group are needed. The process requires judgments about the intensity of preferences for group members and their relative importance in the group. In addition, better models to evaluate morbidity and mortality consequences of decision problems would be helpful. Additional research on structuring and eliciting preferences for impacts over time, especially for nonmonetary impacts, could make a substantial contribution.

Regarding analysis of alternatives, the main research involves developing better computer programs to assist in completing the tasks of decision analysis and better presentation materials and procedures to realize the full potential benefits of decision analyses.

## 8. SUMMARY

Decision analysis embodies a philosophy, some concepts, and an approach to formally and systematically examine a decision problem. It is in no way a substitute for creative, innovative thinking, but rather it promotes and utilizes such efforts to provide important insights into a problem. Philosophically, decision analysis relies on the basis that the desirability of an alternative should depend on two concerns: the likelihood that the alternative will lead to various consequences and the decision maker's preferences for those consequences. Decision analysis addresses these concerns separately and provides the logic and techniques for integrating them.

The foundations of decision analysis are a set of axioms which define the basic constructs, judgmental probability and utility, and the logic of the approach. From these axioms, the fundamental result of decision analysis is derived: the alternative with the highest expected utility should be the most preferred. I, and many other individuals, find these axioms compelling for analyzing decision problems to *prescribe* what alternative a decision maker should choose. It is important to note that any decision making approach which is not consistent with all of the axioms of decision analysis must, by definition, violate at least one of them.

To one who believes in the axioms, the standard for correctness in formally analyzing decision problems must be decision analysis. This does not mean other approaches are not more appropriate in some cases, but that the additional assumptions necessary for these other approaches are important to understand and appraise. Often, however, other seemingly competitive approaches are not in conflict with the axioms of decision analysis. For instance, in cases where there are no uncertainties in describing the possible implications of each alternative and where the utility function (i.e., objective function) is linear, linear programming does not violate the axioms of decision analysis. In such cases, it could be considered a tool of decision analysis, with the big advantage of effectively evaluating an infinite number of alternatives and selecting the best one.

A unique aspect of the decision analysis formulation is the theory and procedures developed to formally introduce and process subjective judgments in the evaluation of alternatives. Professional and value judgments are clearly an important part of the major problems facing our society. With problems concerning abortion, the desirability of capital punishment, or the treatment of terrorists, professional judgments about the likelihoods of various consequences resulting from each alternative and the value judgments required to evaluate such alternatives must be made. In decisions concerning inflation or the energy situation of the country, judgments must somehow be formulated about the likely effects of various policies. Value tradeoffs must be made between inflation rates and unemployment or between the energy available for personal use (i.e., comfort) and national dependence on foreign fuels. In many cases, to neglect such features misses the essence of the problem altogether.

Experience in using decision analysis indicates that knowledgeable professionals, industry executives, and government officials are willing to address the difficult professional judgments and value questions necessary to focus meaningfully on the characteristics of complex decision problems. However, most analyses of important decision problems have left the incorporation of judgments and values to informal procedures with unidentified assumptions and to the intuition of the decision makers. What has been lacking is not information but a framework to articulate and inte-

grate the values and professional judgments of decision makers and experts with the existing data to examine the overall implications of alternative courses of action. Decision analysis provides this framework.

### **Acknowledgement**

David E. Bell, James S. Dyer, Craig W. Kirkwood, D. Warner North, William P. Pierskalla, Stephen M. Pollack, Richard G. Richels, Howard Raiffa, Robert L. Winkler, and Detlof von Winterfeldt made many helpful suggestions on earlier drafts of this article. This work was supported by The Office of Naval Research under contract N00014-81-C-0536.

### **APPENDIX. THE AXIOMS OF DECISION ANALYSIS**

A unique feature of decision analysis is that it has an axiomatic foundation. The axioms provide the rationale and theoretical feasibility for the "divide and conquer" approach of decision analysis. In Section 3, decision analysis was decomposed into four steps:

1. Structure the decision problem;
2. Assess possible impacts of each alternative;
3. Determine preferences (values) of decision makers; and
4. Evaluate and compare alternatives.

Axioms corresponding to Steps 1 through 3 state conditions under which it is feasible to obtain the necessary information for a decision analysis decomposed in this manner. Axioms corresponding to Step 4 provide the substance for aggregating the information in the preceding steps to evaluate the alternatives.

To facilitate understanding, the axioms of decision analysis are stated here in an informal and intuitive manner. The complete sense of the axioms is preserved although they are not technically precise. A formal statement of the axioms is found in Pratt, Raiffa, and Schlaifer [1964]. In the following, Axioms 1a and b pertain to Step 1 of the decision analysis methodology, Axiom 2 pertains to Step 2, and so on.

**Axiom 1a—Generation of Alternatives.** At least two alternatives can be specified.

For each of the alternatives, there will be a number of possible consequences which might result if that alternative is followed.

**Axiom 1b—Identification of Consequences.** Possible consequences of each alternative can be identified.

In identifying consequences, it may be useful to generate an objectives hierarchy indicating the domain of potential consequences in the problem. Attributes can be specified to provide evaluation scales necessary to indicate the degree to which each objective is achieved.

**Axiom 2—Quantification of Judgment.** The relative likelihoods (i.e., probabilities) of each possible consequence that could result from each alternative can be specified.

As discussed in Section 3, there are a number of procedures to assist in specifying relative likelihoods. Such probabilistic estimates are based on available data, information collected, analytical or simulation models, and assessment of experts' judgments.

**Axiom 3—Quantification of Preference.** The relative desirability (i.e., utility) for all the possible consequences of any alternative can be specified.

The preferences which should be quantified in a decision problem are those of the decision makers. It is very helpful if one can assess these preferences directly from the decision maker or decision makers. However, for many problems other individuals have a responsibility for recommending alternatives to the decision makers. In such problems, those individuals may have a responsibility for articulating an appropriate preference structure.

**Axiom 4a—Comparison of Alternatives.** If two alternatives would each result in the same two possible consequences, the alternative yielding the higher chance of the preferred consequence is preferred.

**Axiom 4b—Transitivity of Preferences.** If one alternative is preferred to a second alternative and if the second alternative is preferred to a third alternative, then the first alternative is preferred to the third alternative.

**Axiom 4c—Substitution of Consequences.** If an alternative is modified by replacing one of its consequences with a set of consequences and associated probabilities (i.e., a lottery) that is indifferent to the consequence being replaced, then the original and the modified alternatives should be indifferent.

Axiom 4a is necessary to indicate how various alternatives should be compared. Axioms 4b and 4c are often referred to as consistency axioms. Axiom 4c allows one to reduce complex alternatives involving a variety of possible consequences to simple alternatives referred to in Axiom 4a. It is then easy to compare alternatives. Axiom 4b is necessary to include comparisons of more than two alternatives.

The main result of these axioms is that the expected utility of an alternative is the indication of its desirability. Alternatives with higher expected utilities should be preferred to those with lower expected utilities. The probabilities and utilities necessary to calculate expected utility emerge as distinct items. Information about each must be gathered in conducting a decision analysis. However, the axioms themselves provide little guidance about how to obtain this information. This is discussed in Sections 3 and 4 on the methodology and practice of decision analysis.

Decision analysis does not require either a single decision maker or identifiable decision makers. It requires an orientation toward the decision to be made and individuals able and willing to provide information essential to that decision. The essential assumptions in this regard are those given in axioms corresponding to Steps 1, 2, and 3. What is assumed is that the information required by those assumptions can be obtained in a useful manner. A decision analysis which structured and analyzed a decision problem without any interaction with or knowledge of the "decision makers" could provide a tremendous amount of insight—the product of decision analysis—to them if they saw the analysis. Recognition of this misconception is important because it has often been stated that an implicit assumption of decision analysis is that the decision maker is a single individual and can be identified. It is further claimed that this assumption is invalid for essentially all important decision problems. Hence, decision analysis is at best an interesting theoretical exercise with little practical value. While it may be easier to structure a decision problem and provide critical information by interacting with the identified decision makers, many critical problems do not afford this luxury. It is not essential for constructive decision analysis to occur.

## REFERENCES

- M. Alpert and H. Raiffa, "A Progress Report on the Training of Probability Assessors," in *Judgment Under Uncertainty: Heuristics and Biases*, D. Kahneman, P. Slovic, and A. Tversky, eds., Cambridge University Press, New York, 1981.
- K. J. Arrow, *Social Choice and Individual Values*, 2nd ed., Wiley, New York, 1963.
- J. R. Beley, P. Fleischauer, R. L. Keeney, C. W. Kirkwood and A. Sicherman, "Decision Framework for Technology Choice—Volume 1: A Case Study of One Utility's Coal-Nuclear Choice," EA-2153, Research Project 1433-1, Interim Report, Electric Power Research Institute, Palo Alto, California, 1981.
- D. E. Bell, "A Decision Analysis of Objectives for a Forest Pest Problem," in *Conflicting Objectives in Decisions*, D. E. Bell, R. L. Keeney and H. Raiffa, eds., Wiley (Interscience), New York, 1977a.
- D. E. Bell, "A Utility Function for Time Streams Having Interperiod Dependencies," *Operations Research*, 25, pp. 448-458, 1977b.
- D. E. Bell, "Consistent Assessment Procedures Using Conditional Utility Functions," *Operations Research*, 27, pp. 1054-1066, 1979a.
- D. E. Bell, "Multiattribute Utility Functions: Decompositions Using Interpolation," *Management Science*, 25, pp. 744-753, 1979b.
- D. Bernoulli, "Exposition of a New Theory on the Measurement of Risk," *Commentarii Academiae Scientiarum Imperialis Petropolitanae*, 5, pp. 175-192, 1738. Translated by L. Sommer, *Econometrica*, 22, pp. 23-36, 1954.
- D. Blackwell and M. A. Girshick, *Theory of Games and Statistical Decisions*, Wiley, New York, 1954.
- S. E. Bodily, "Analysis of Risks to Life and Limb," *Operations Research*, 28, pp. 156-175, 1980.
- D. Boyd, "Interactive Preference Assessment for Decision with Multiattribute Outcomes," Fourth Research Conference on Subjective Probability, Utility, and Decision Making, Rome, Italy, 1973.
- D. W. Boyd, C. E. Clark, Jr., and D. W. North, "Analysis of a Government Initiative to Accelerate Commercial Acceptance of Solar Photovoltaic Systems," *Journal of the Operational Research Society*, 32, to appear 1982.
- R. V. Brown, "Do Managers Find Decision Theory Useful?," *Harvard Business Review*, 48, pp. 78-89, 1970.
- R. V. Brown, A. S. Kahr and C. Peterson, *Decision Analysis for the Manager*, Holt, New York, 1974.
- E. G. Cazalet, C. E. Clark and T. W. Keelin, "Costs and Benefits of Over Under Capacity in Electric Power System Planning," EA-927, Research Project 1107, Electric Power Research Institute, Palo Alto, California, 1978.
- D. M. Crawford, B. C. Huntzinger and C. W. Kirkwood, "Multiobjective Decision Analysis for Transmission Conductor Selection," *Management Science*, 24, pp. 1700-1709, 1978.
- D. Davidson, P. Suppes and S. Siegel, *Decision Making: An Experimental Approach*, Stanford University Press, Stanford, California, 1957.
- G. Debreu, "Topological Methods in Cardinal Utility Theory," in *Mathematical Methods in the Social Sciences, 1959*, K. J. Arrow, S. Karlin and P. Suppes, eds., Stanford University Press, Stanford, California, 1960.
- B. DeFinetti, "La prevision: ses lois logiques, ses sources subjectives," *Annales de l'Institut Henri Poincaré*, 7, pp. 1-68, 1937. English translation by H. E. Kyburg, Jr., in H. E. Kyburg, Jr., and H. E. Smokler, eds., *Studies in Subjective Probability*, Wiley, New York, 1964.
- R. de Neufville and R. L. Keeney, "Use of Decision Analysis in Airport Development in Mexico City," in *Analysis of Public Systems*, A. W. Drake, R. L. Keeney and P. M. Morse, eds., M.I.T. Press, Cambridge, Massachusetts, 1972.
- J. S. Dyer and R. F. Miles, Jr., "An Actual Application of Collective Choice Theory to the Selection of Trajectories for the Mariner Jupiter/Saturn 1977 Project," *Operations Research*, 24, pp. 220-244, 1976.
- J. S. Dyer and R. K. Sarin, "Measurable Multiattribute Value Functions," *Operations Research*, 27, pp. 810-822, 1979.
- W. Edwards, "The Theory of Decision Making," *Psychological Bulletin*, 51, pp. 380-417, 1954.
- W. Edwards, "Conservatism in Human Information Processing," in *Formal Representations of Human Judgment*, B. Klein-Muntz, ed., Wiley, New York, 1968.

- W. Edwards, "Reflections on and Criticisms of a Highly Political Multiattribute Utility Analysis," in *Mathematical Frontiers of Behavioral and Policy Sciences*, L. Cobb and R. W. Thrall, eds., Westview Press, Boulder, Colorado, 1980.
- H. J. Einhorn and R. M. Hogarth, "Behavioral Decision Theory: Processes of Judgment and Choice," *Annual Review of Psychology*, 32, pp. 53-88, 1981.
- P. H. Farquhar, "A Fractional Hypercube Decomposition Theorem for Multiattribute Utility Functions," *Operations Research*, 23, pp. 941-967, 1975.
- P. H. Farquhar and P. C. Fishburn, "Equivalences and Continuity in Multivalent Preference Structures," *Operations Research*, 29, pp. 282-293, 1981.
- W. Fellner, *Probability and Profit: A Study of Economic Behavior Along Bayesian Lines*, Richard D. Irwin, Homewood, Illinois, 1965.
- G. W. Fischer, "Multidimensional Utility Models for Risky and Riskless Choice," *Organizational Behavior and Human Performance*, 17, pp. 127-146, 1976.
- G. W. Fischer, "Utility Models for Multiple Objective Decisions: Do They Accurately Represent Human Preferences," *Decision Science*, 10, pp. 451-479, 1979.
- B. Fischhoff, "Clinical Decision Analysis," *Operations Research*, 28, pp. 28-43, 1980.
- P. C. Fishburn, *Decision and Value Theory*, Wiley, New York, 1964.
- P. C. Fishburn, "Independence in Utility Theory with Whole Product Sets," *Operations Research*, 13, pp. 28-45, 1965.
- P. C. Fishburn, "Methods of Estimating Additive Utilities," *Management Science*, 13, pp. 435-453, 1967.
- P. C. Fishburn, *Utility Theory for Decision Making*, Wiley, New York, 1970.
- P. C. Fishburn, *The Theory of Social Choice*, Princeton University Press, Princeton, New Jersey, 1973.
- P. C. Fishburn, "Von Neumann-Morgenstern Utility Functions on Two Attributes," *Operations Research*, 22, pp. 35-45, 1974.
- M. Friedman and L. J. Savage, "The Expected-Utility Hypothesis and the Measurability of Utility," *Journal of Political Economy*, 60, pp. 463-474, 1952.
- P. C. Gardiner and W. Edwards, "Public Values: Multi-attribute Utility Measurement for Social Decision Making," in *Human Judgment and Decision Processes*, M. F. Kaplan and S. Swartz, eds., Academic Press, New York, 1975.
- R. J. Gilbert and R. G. Richels, "Reducing Uranium Resource Uncertainty: Is it Worth the Cost?", *Resources and Energy*, 3, to appear, 1981.
- C. J. Grayson, *Decision Under Uncertainty: Drilling Decisions by Oil and Gas Operators*, Division of Research, Harvard Business School, Boston, Massachusetts, 1960.
- K. R. Hammond, G. H. McClelland and J. Mumpower, *Human Judgment and Decision Making*, Praeger Publishers, New York, 1980.
- J. C. Harsanyi, "Cardinal Welfare, Individualistic Ethics, and Interpersonal Comparisons of Utility," *Journal of Political Economy*, 63, pp. 309-321, 1955.
- R. M. Hogarth, "Cognitive Processes and the Assessment of Subjective Probability Distributions," *Journal of the American Statistical Association*, 70, pp. 271-289, 1975.
- C. S. Holling, ed., *Adaptive Environmental Assessment and Management*, Wiley (Interscience), New York, 1978.
- C. A. Holloway, *Decision Making Under Uncertainty*, Prentice-Hall, Englewood Cliffs, New Jersey, 1979.
- R. A. Howard, "Decision Analysis: Applied Decision Theory," *Proceedings of the Fourth International Conference on Operational Research*, Boston, Massachusetts, 1966.
- R. A. Howard, "Life and Death Decision Analysis," Research Report No. EES DA-79-2, Department of Engineering-Economic Systems, Stanford University, Stanford, California, 1979.
- R. A. Howard, J. E. Matheson and D. W. North, "The Decision to Seed Hurricanes," *Science*, 176, pp. 1191-1202, 1972.
- G. P. Huber, "Methods for Quantifying Subjective Probabilities and Multiattribute Utilities," *Decision Science*, 5, pp. 430-458, 1974.



- B. R. Judd, "Decision Analysis of Electrical Capacity Expansion," *Lawrence Symposium on Systems and Decision Sciences 1977* Western Periodicals, North Hollywood, California, 1978.
- H. Jungermann, "Structural Modeling of Decision Problems," Institute for Psychology, Technical University of Berlin, 1980.
- D. Kahneman and A. Tversky, "Prospect Theory: An Analysis of Decision Under Risk," *Econometrica*, 47, pp. 263-291, 1979.
- G. J. Kaufman, *Statistical Decision and Related Techniques in Oil and Gas Exploration*, Prentice-Hall, Englewood Cliffs, New Jersey, 1963.
- G. J. Kaufman and H. Thomas, eds., *Modern Decision Analysis*, Penguin Books, New York, 1977.
- D. L. Keefer and C. W. Kirkwood, "A Multiobjective Decision Analysis: Budget Planning for Product Engineering," *Journal of the Operational Research Society*, 29, pp. 435-442, 1978.
- T. W. Keelin, "A Parametric representation of Additive Value Functions," *Management Science*, 27, pp. 1200-1208, 1981.
- R. L. Keeney, "Examining Corporate Policy Using Multiattribute Utility Analysis," *Sloan Management Review*, 17, pp. 63-76, 1975.
- R. L. Keeney, "Decision Analysis," in *Handbook of Operations Research*, J. J. Moder and S. E. Elmaghraby, eds., Van Nostrand Reinhold, New York, 1978.
- R. L. Keeney, "Evaluating Alternatives Involving Potential Fatalities," *Operations Research*, 28, pp. 188-205, 1980a.
- R. L. Keeney, *Siting Energy Facilities*, Academic Press, New York, 1980b.
- R. L. Keeney, "Measurement Scales for Quantifying Attributes," *Behavioral Science*, 26, pp. 29-36, 1981.
- R. L. Keeney and C. W. Kirkwood, "Group Decision Making Using Cardinal Social Welfare Functions," *Management Science*, 22, pp. 430-437, 1975.
- R. L. Keeney and K. Nair, "Selecting Nuclear Power Plant Sites in the Pacific Northwest Using Decision Analysis," in *Conflicting Objectives in Decisions*, D. E. Bell, R. L. Keeney and H. Raiffa, eds., Wiley (Interscience), New York, 1977.
- R. L. Keeney and H. Raiffa, "A Critique of Formal Analysis in Public Decision Making," in *Analysis of Public Systems*, A. W. Drake, R. L. Keeney and P. M. Morse, eds., M.I.T. Press, Cambridge, Massachusetts, 1972.
- R. L. Keeney and H. Raiffa, *Decisions with Multiple Objectives*, Wiley, New York, 1976.
- R. L. Keeney and A. Sicherman, "An Interactive Computer Program for Assessing and Analyzing Preferences Concerning Multiple Objectives," *Behavioral Science*, 21, pp. 173-182, 1976.
- C. W. Kirkwood and S. M. Pollock, "Bounding Probabilities for Scenarios with Multiple Attributes," Department of Industrial and Operations Engineering, University of Michigan, Ann Arbor, 1980.
- C. W. Kirkwood and R. K. Sarin, "Preference Conditions for Multiattribute Value Functions," *Operations Research*, 28, pp. 225-232, 1980.
- T. C. Koopmans, "Stationary Ordinal Utility and Impatience," *Econometrica*, 28, pp. 287-309, 1960.
- D. H. Krantz, "Conjoint Measurement: The Luce-Tukey Axiomatization and Some Extensions," *Journal of Mathematical Psychology*, 1, pp. 248-277, 1964.
- D. H. Krantz, R. D. Luce, P. Suppes and A. Tversky, *Foundations of Measurement*, 1, Academic Press, New York, 1971.
- J. P. Krischer, "An Annotated Bibliography of Decision Analytic Applications to Health Care," *Operations Research*, 28, pp. 97-113, 1980.
- K. J. Lancaster, "An Axiomatic Theory of Consumer Time Preference," *International Economic Review*, 4, pp. 221-231, 1963.
- J. W. Lathrop and S. R. Watson, "Decision Analysis for the Evaluation of Risk in Nuclear Waste Management," *Journal of the Operational Research Society*, 33, to appear, 1982.
- I. H. LaValle, "On Cash Equivalents and Information Evaluation in Decisions Under Uncertainty," *Journal of the American Statistical Association*, 63, pp. 252-290, 1968.
- I. H. LaValle, *Fundamentals of Decision Analysis*, Holt, New York, 1978.

- W. W. L. Lee, *Decisions in Marine Mining: The Role of Preferences and Tradeoffs*, Ballinger Publishers, Cambridge, Massachusetts, 1979.
- D. Longbottom and G. Wade, "An Investigation into the Application of Decision Analysis in United Kingdom Companies," *Omega*, 1, pp. 207-215, 1973.
- R. D. Luce and J. W. Tukey, "Simultaneous Conjoint Measurement: A New Type of Fundamental Measurement," *Journal of Mathematical Psychology*, 1, pp. 1-27, 1964.
- J. F. Magee, "Decision Trees for Decision Making," *Harvard Business Review*, 42, No. 4, pp. 126-138, 1964a.
- J. F. Magee, "How to Use Decision Trees in Capital Investment," *Harvard Business Review*, 42, No. 5, pp. 79-96, 1964b.
- G. Majone and E. W. Quade, eds., *Pitfalls of Analysis*, Wiley, Chichester, England, 1980.
- A. S. Manne and R. G. Richels, "A Decision Analysis of the U.S. Breeder Reactor Program," *Energy*, 3, pp. 747-767, 1978.
- J. E. Matheson, "Decision Analysis Practice: Examples and Insights," *Proceedings of the Fifth International Conference on Operational Research*, Venice, Italy, 1969.
- M. W. Merkhofer, "The Value of Information Given Decision Flexibility," *Management Science*, 23, pp. 716-727, 1977.
- R. F. Meyer, "On the Relationship Among the Utility of Assets, the Utility of Consumption, and Investment Strategy in an Uncertain, but Time Invariant World," in *OR 69, Proceedings of the Fifth International Conference on Operational Research*, J. Lawrence, ed., Tavistock Publishing, London, 1970.
- R. F. Meyer, "State-dependent Time Preference," in *Conflicting Objectives in Decisions*, D. E. Bell, R. L. Keeney and H. Raiffa, eds., Wiley (Interscience), New York, 1977.
- A. C. Miller, M. W. Merkhofer, R. A. Howard, J. E. Matheson and T. R. Rice, "Development of Automated Aids for Decision Analysis," SRI Report 3309, SRI International, Menlo Park, California, 1976.
- P. G. Moore and H. Thomas, *The Anatomy of Decisions*, Penguin Books, New York, 1976.
- P. A. Morris, "Combining Expert Judgments: A Bayesian Approach," *Management Science*, 23, pp. 679-693, 1977.
- F. Mosteller and P. Nogee, "An Experimental Measurement of Utility," *Journal of Political Economy*, 59, pp. 371-404, 1951.
- A. H. Murphy and R. L. Winkler, "Reliability of Subjective Probability Forecasts of Precipitation and Temperature," *Applied Statistics*, 26, pp. 41-47, 1977.
- D. W. North, F. Offensend, and C. Smart, "Planning Wildfire Protection for the Santa Monica Mountains," *Fire Journal*, 69, No. 1, 1975.
- D. W. North and M. W. Merkhofer, "A Methodology for Analyzing Emission Control Strategies," *Computers and Operations Research*, 3, pp. 185-207, 1976.
- J. S. Pliskin, D. S. Shepard and M. C. Weinstein, "Utility Functions for Life Years and Health Status," *Operations Research*, 28, pp. 206-224, 1980.
- R. A. Pollak, "Additive von Neumann-Morgenstern Utility Functions," *Econometrica*, 35, pp. 485-494, 1967.
- J. W. Pratt, "Risk Aversion in the Small and in the Large," *Econometrica*, 32, pp. 122-136, 1964.
- J. W. Pratt, H. Raiffa and R. O. Schlaifer, "The Foundations of Decision Under Uncertainty: An Elementary Exposition," *Journal of the American Statistical Association*, 59, pp. 353-375, 1964.
- J. W. Pratt, H. Raiffa and R. O. Schlaifer, *Introduction to Statistical Decision Theory*, McGraw-Hill, New York, 1965.
- H. Raiffa, *Decision Analysis*, Addison-Wesley, Reading, Massachusetts, 1968.
- H. Raiffa, "Preference for Multi-attributed Alternatives," RM-5868-DOT/RC, The Rand Corporation, Santa Monica, California, 1969.
- H. Raiffa and R. O. Schlaifer, *Applied Statistical Decision Theory*, Harvard Business School, Division of Research, Boston, Massachusetts, 1961. Reprinted by the M.I.T. Press, Cambridge, Massachusetts, 1968.
- F. P. Ramsey, "Truth and Probability," in *The Foundations of Mathematics and Other Logical Essays*, by F. P. Ramsey, Harcourt, Brace and Co., New York, 1931. Reprinted in *Studies in Subjective Probability*, H. E. Kyburg and H. E. Smokler, eds., Wiley, New York, 1964.

- W. R. Rousseau and J. E. Matheson, "Computer Programs for Decision Analysis," Stanford Research Institute, Menlo Park, California, 1967.
- R. K. Sarin, "A Sequential Approach to Cross-Impact Analysis," *Futures*, 10, pp. 53-62, 1978.
- R. K. Sarin, "Ranking of Multiattribute Alternatives with an Application to Coal Power Plant Siting," in *Multiple Criteria Decision Making—Theory and Application*, G. Fandel and T. Gal, eds., Springer-Verlag, Berlin, 1980.
- L. J. Savage, *The Foundations of Statistics*, Wiley, New York, 1954.
- R. O. Schlaifer, *Probability and Statistics for Business Decisions*, McGraw-Hill, New York, 1959.
- R. O. Schlaifer, *Manual of Cases on Decision Under Uncertainty With Analyses and Teaching Notes*, McGraw-Hill, New York, 1968.
- R. O. Schlaifer, *Analysis of Decisions Under Uncertainty*, McGraw-Hill, New York, 1969.
- R. O. Schlaifer, *Computer Programs for Elementary Decision Analysis*, Division of Research, Harvard Business School, Boston, Massachusetts, 1971.
- D. A. Seaver, D. von Winterfeldt and W. Edwards, "Eliciting Subjective Probability Distributions on Continuous Variables," *Organizational Behavior and Human Performance*, 21, pp. 379-391, 1978.
- F. Seo and M. Sakawa, "An Evaluation method for Environmental-systems Planning: An Alternative Utility Approach," *Environment and Planning*, 11, pp. 149-168, 1979.
- F. Seo, M. Skaya, H. Takahashi, K. Nakagami and H. Horiyama, "An Interactive Computer Program for Multiattribute Utility Analysis," GE18-1890-0, Tokyo Scientific Center, IBM, Tokyo, Japan, 1978.
- P. Slovic and S. Lichtenstein, "Comparison of Bayesian and Regression Approaches to the Study of Information Processing in Judgment," *Organizational Behavior and Human Performance*, 6, pp. 649-744, 1971.
- R. D. Smallwood and P. A. Morris, "A Task Force Decision Analysis," *Operations Research*, 18, pp. 60-80, 1980.
- C. S. Spetzler, "The Development of a Corporate Risk Policy for Capital Investment Decision," *IEEE Systems Science and Cybernetics*, SSC-4, pp. 279-300, 1968.
- C. S. Spetzler and C-A. S. Staël von Holstein, "Probability Encoding in Decision Analysis," *Management Science*, 22, pp. 340-358, 1975.
- C-A. S. Staël von Holstein and J. E. Matheson, "A Manual for Encoding Probability Distributions," SRI Report 7078, SRI International, Menlo Park, California, 1979.
- W. G. Stillwell, F. H. Barron and W. Edwards, "Evaluating Credit Applications: A Validation of Multiattribute Utility Techniques Against a Real World Criterion," SSRI Research Report 80-1, Social Science Research Institute, University of Southern California, Los Angeles, California, 1980.
- Synfuels Interagency Task Force, *Recommendations for a Synthetic Fuels Commercialization Policy*, U.S. Government Printing Office, Stock No. 041-001-00111-3, 1975.
- H. Tamura and Y. Nakamura, "Decompositions of Multiattribute Utility Functions Based on a New Concept of Convex Dependence," Conference of IEEE Systems, Man and Cybernetics Society, Japan, 1978.
- M. Tribus, *Rational Descriptions, Decisions, and Designs*, Pergamon Press, New York, 1969.
- A. Tversky and D. Kahneman, "Judgment Under Uncertainty: Heuristics and Biases," *Science*, 185, pp. 1124-1131, 1974.
- A. Tversky and D. Kahneman, "The Framing of Decisions and the Psychology of Choice," *Science*, 211, pp. 453-458, 1981.
- J. W. Ulvila and W. D. Snider, "Negotiation of International Oil Tanker Standards: An Application of Multiattribute Value Theory," *Operations Research*, 28, pp. 81-96, 1980.
- J. von Neumann and O. Morgenstern, *Theory of Games and Economic Behavior*, 2nd ed., Princeton University Press, Princeton, New Jersey, 1947.
- D. von Winterfeldt, "Setting standards for Offshore Oil Discharges: A Regulatory Decision Analysis," *Operations Research*, 30, to appear, 1982.
- D. von Winterfeldt, "Structuring Decision Problems for Decision Analysis," *Acta Psychologica*, 45, pp. 71-93, 1980.
- A. Wald, *Statistical Decision Functions*, Wiley, New York, 1950.

- R. L. Winkler, "The Assessment of Prior Distributions in Bayesian Analysis," *Journal of the American Statistical Association*, 62, pp. 776-800, 1967a.
- R. L. Winkler, "The Quantification of Judgment: Some Methodological Suggestions," *Journal of the American Statistical Association*, 62, pp. 1105-1120, 1967b.
- R. L. Winkler, "Scoring Rules and the Evaluation of Probability Assessors," *Journal of the American Statistical Association*, 64, pp. 1073-1078, 1969.
- R. L. Winkler, *Introduction to Bayesian Inference and Decision*, Holt, New York, 1972.
- R. L. Winkler, "Combining Probability Distributions from Dependent Information Sources," *Management Science*, 27, pp. 479-488, 1981.